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The Magnus or Robins effect on rotating spheres

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Robins showed in 1742 that a transverse aerodynamic force on a rotating sphere could be detected by suspending it as a pendulum. Differences of periodic time in conical pendulum motion with spin and orbit parallel and opposed have been found to give a reasonably accurate measure of the lift coefficient, and the results shown extend knowledge of the effect down to a Reynolds number of 2×10^3 and up to a ratio of 12 between the peripheral and translational velocities.

1. Introduction

Robins must be accounted one of the founders of aerodynamics in virtue of his measurements of air resistance. The ballistic pendulum and the whirling arm were his own inventions, and these he employed with great skill and care in two complementary investigations: with the first he measured the loss of 'celerity' of musket-balls at transonic speeds up to a Mach number of 1·5, recognizing clearly the existence of a disproportionate rise in drag at speeds in the region of the velocity of sound; with the second device he made direct observations on the resistance of larger spheres at speeds of 10–40 ft s^{-1}, confirming, for this range, the proportionality between resistance and the square of the velocity, as enunciated by Newton (1687). In addition, Robins made several remarkable observations on the drag of other bodies, including the first indication of the effect of aspect ratio.

Finally, he established clear priority in experimental demonstration of the phenomenon commonly known as the Magnus effect. The experiments carried out by Magnus (1853), more than a century after Robins's work, were successful only with rotating cylinders, and were purely qualitative; no measurements or even estimates are recorded for the velocities and forces. The equivalent effect with spheres was certainly known nearly two centuries earlier. Newton (1671/2) had noted how the flight of a tennis ball was affected by spin, and had given an explanation; the motions conspiring to excite a greater reluctancy and reaction implied a greater pressure on that side of the ball which was moving forward faster.

Euler's opposition to the idea is surprising. Even while professing admiration for Daniel Bernoulli's *Hydrodynamica*, to which he considered that Robins paid insufficient respect, Euler (1777) rejected the possibility of an aerodynamic force resulting from spin. 'The want of a perfect roundness' (p. 310) he believed to be the only cause of the deflexion of shot.

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The association of the effect with the name of Magnus was confirmed by the attribution to him by Rayleigh (1877) of 'the true explanation'. Rayleigh nevertheless noted the 'weak step' of the argument, namely that the pressure was greatest on the side where the velocity was least, which could be justified for frictionless fluids only, whereas friction is 'the immediate cause of the whirlpool motion'. Even more questionable, though not apparently challenged, was the supposed experimental demonstration by Magnus that the pressures were not equal on the two sides of the cylinder; the observed movement of quite large vanes was proof only that pressure differences existed across the vanes in consequence of a special and quite different condition of flow resulting from the presence of the vanes.

Since Magnus was unsuccessful in his experiments with spheres, merely guessing that a similar force was generated, it would be only justice if the case of the sphere were to be renamed the Robins effect. The extension of Robins's work on rotating spheres is the main subject of this paper.

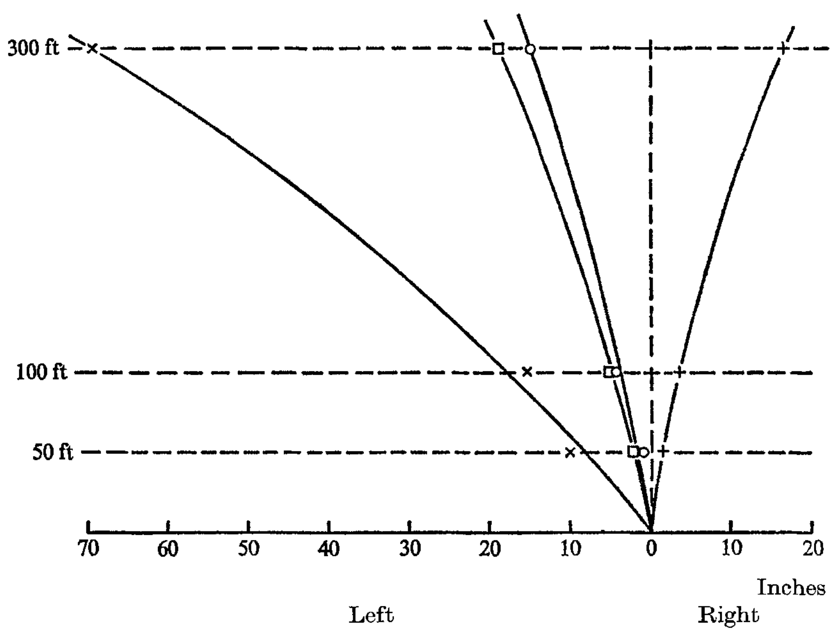

FIGURE 1. The first set of observations by Robins. Five shots being fired in slightly different directions from the same notch, the horizontal distances of the last four tracks are shown relative to the first, as measured on three planes. Screens at 50 ft and 100 ft were of 'exceeding thin paper', and the third plane was a wall at 300 ft.

2. Robins's observations on the effect of spin

Tracing the trajectories of musket-balls on two thin paper screens and a wall, Robins (1805, p. 210) found unmistakable evidence of curvature in either direction in the horizontal plane; his belief that this was due to random components of spin about a vertical axis was confirmed by deliberately imparting spin about a known axis by means of a 'crooked piece'; the barrel being bent to the left, the ball was forced into contact with the right side of the bore, and the shots were curved to the right.

The absolute alignment of the lines of reference on the three planes was not considered essential for establishing the principle in the presence of other observers. The mark of the first shot being taken as the datum on each screen, deflexions of subsequent shots were measured from these, and the recorded sets of differences are plotted in figures 1 and 2; for the second group a more loosely fitting ball was used. The evidence is insufficient for statistical treatment, but the average curvature of the five shots in the second group is zero, if the first shot

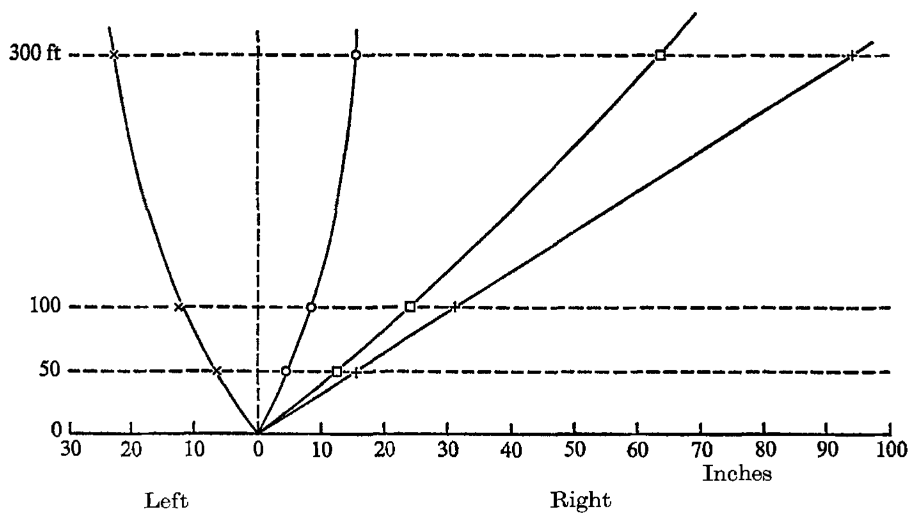

FIGURE 2. The second of the two sets of observations. If the mean curvature of these five shots from the straight barrel is assumed to be zero, then, taking each in turn as the reference shot used by Robins, the curvature of the 'crooked-piece' shot has the following values: 6·1, 6·3, 10·5, 3·3 or $4{\cdot}3 \times 10^{-5}\,\text{ft}^{-1}$.

of the group is taken to have a curvature to the right of $0{\cdot}2 \times 10^{-5}\,\text{ft}^{-1}$. Although the behaviour of the later shots with a barrel curved 3 or 4° to the left evidently satisfied the observers, only one is recorded. Robins writes: "...notwithstanding the bend of the piece to the left, the bullet itself might be expected to incurvate towards the right; and this, upon trial, did most remarkably happen". The one recorded shot had a curvature to the right of $6{\cdot}3 \times 10^{-5}\,\text{ft}^{-1}$ relative to one of the five shots of figure 2, unfortunately not specified. Unless one makes the unlikely assumption that the average curvature of the five shots was really not zero but markedly to the left, then the crooked-piece shot must have been more curved to the right than even the most unfavourable possible reference shot was curved to the left. The five possible values quoted in the caption to figure 2 give the absolute curvature of the crooked-piece shot on the assumption that the average curvature of the five straight-barrel shots was zero.

A rough estimate can be made of the spin imparted to the lead ball, assuming it to be travelling at the full muzzle-velocity U before reaching the bend; as Robins bent only the last 3 or 4 in. of the barrel, this is reasonable. If the ball has mass m and radius a, and if the radius of curvature of the barrel is R, friction, with a coefficient μ, gives an accelerating moment $\mu m U^2 a/R$. The angular velocity acquired while the barrel bends through an angle θ is

$$\frac{\text{moment} \times \text{time}}{\text{moment of inertia}} = \frac{(\mu m U^2 a/R)(R\theta/U)}{\frac{2}{5}ma^2} = \frac{5}{2}\frac{\mu U\theta}{a}.$$

Hence, whether the curvature of the barrel is uniform or not, the ratio of peripheral velocity to translational velocity is given by

$$V/U = \tfrac{5}{2}\mu\theta.$$

Taking the higher figure of 4° for the bend, and a value $\mu = 0{\cdot}2$, this gives $V/U = 0{\cdot}035$, which will be seen to be consistent with a curvature of $6 \times 10^{-5}\,\text{ft}^{-1}$, as far as available aerodynamic evidence allows.

No direct experiments are known to have been made on the 'lift' of rotating spheres at speeds approaching those of musket-balls. The measurements of Maccoll (1928) on a smooth sphere over a range of V/U up to 7 were made with values of U up to $34\,\text{ft}\,\text{s}^{-1}$. These are consistent with observations by Davies (1949) using rough as well as smooth golf balls at $U = 105\,\text{ft}\,\text{s}^{-1}$. Both experimenters show negative lift – that is, a reversed Magnus or Robins force – for smooth balls at values of V/U up to about $\frac{1}{2}$. This includes, therefore, all the spins imparted by the straight or curved barrels in Robins's experiments, but if the explanation offered by Davies for this reversal of sign of the lift is correct, then, at the higher velocities of a musket-ball, rough and smooth balls would be expected to experience similar forces.

Davies argued from the known difference, for smooth non-rotating spheres, of the pressure distribution at Reynolds numbers above and below a critical value of about 3×10^5. The turbulent boundary layer following the surface further round at the higher speeds, and breakaway occurring before maximum width at the lower speeds, the region of reduced pressure is developed to more nearly that of potential flow when the Reynolds number is above the critical value. If we then consider a smooth ball in an air stream such that the Reynolds number would be close to the critical value in the absence of spin, and if we further suppose the ball to have angular velocity, the flow on the side on which the surface is moving against the stream corresponds to a Reynolds number above the critical value, while that on the opposite side has the characteristics of flow at less than the critical value. The reduction of pressure on the side moving against the stream may then be sufficiently developed to outweigh the less perfect development of the normally dominant low-pressure system. For moderate speeds of rotation a reversal of the usual aerodynamic force might therefore be expected.

Davies used only one wind speed, and the overall Reynolds number was $0{\cdot}9 \times 10^5$. The observations of Maccoll show a smaller negative lift for

$$R = 1{\cdot}1 \times 10^5 \quad \text{than for } R = 0{\cdot}9 \times 10^5.$$

The Reynolds number for musket-balls near the beginning of their flight, namely about 5×10^5, may be presumed to be well beyond the limited range in which smooth-sphere lift-reversal occurs. If, therefore, we use Davies's curve for the lateral force on a rough ball, and take a mean lift coefficient of $C_L = 0{\cdot}080$, corresponding to $V/U = 0{\cdot}035$, we find that the trajectory should have a mean curvature of $5{\cdot}6 \times 10^{-5}\,\mathrm{ft}^{-1}$. (A lateral force proportional to U^2, as is implied by a constant C_L, gives a circular path with a radius independent of velocity. U will, of course, fall by 30 % or 40 % in 300 ft, but V decreases also, so that the change in C_L need not be great. Further refinement of the calculation is not justified here.)

In support of his belief in an aerodynamic force as the explanation of the deflexions of musket-balls, Robins reports another experiment with very much lower velocities. A wooden ball of $4\frac{1}{2}$ in. diameter on a twisted double string of 8 or 9 ft length was set in motion as a pendulum. The subsequent rotation of the ball was accompanied by a rotation of the plane of the swing, in the same direction as the rotation of the ball, and this continued while the string was twisting again, and the rotational speed decreasing. The only quantitative observation recorded by Robins is that the plane of the swing could change direction by as much as 90°. He regarded this as "incontestable proof that if any bullet, besides its progressive motion, hath a whirl around its axis; it will be deflected in the manner here described". In this he was not quite correct. The purely gyroscopic action of the ball on its string makes a contribution to the precession of the swing which is of comparable magnitude to that of the aerodynamic force. Nevertheless the experiment is of interest in itself, and the complementary notion of the conical pendulum having a period dependent on its 'whirl' will be seen to give useful observations.

3. The continuation of Robins's experiments

A teaching-laboratory project was developed from Robins's spinning sphere pendulum. A ball with a slightly roughened surface, having a mass of 100 g and diameter 6·4 cm, was suspended by a nylon thread of varying length from the spindle of an electric motor, and two types of observation were made.

The precession of a simple pendulum, as observed by Robins, was treated as a case of the rotation of the axes in an elliptical orbit. Since the flow pattern around the sphere is continually changing in such orbits, these measurements do not permit the derivations of aerodynamic information of any value. The rate of precession depends markedly on the eccentricity of the ellipse, and this is attributable in part to the gyroscopic effect; as no theoretical treatment of this effect is known to be in print, the results are only reported briefly.

The more nearly the orbits approach to a pure circle, the more nearly the flow approaches a steady state, and, while the concept of precession of axes becomes meaningless, the change of period of the conical pendulum due to spin suggested itself as a measure of the aerodynamic lift. This idea proved to be moderately profitable, and the coefficients derived by this means extend to a higher ratio of peripheral to translational velocity than any previously reported.

3.1. *Precession of elliptical orbits*

Photographic recording of the orbits was facilitated by reflexions from a very small ball of crumpled aluminium foil at the junction of ball and thread, ball and background being dark. The shutter was opened for every fifth or tenth orbit.

Figure 3 (plate 1) shows a typical record. The eccentricity of the orbits changed slowly with time in a somewhat irregular manner. The rate of precession depended only slightly on the amplitude of the orbits; a value of 0·15 rad was chosen as a representative angle of the major half-axes, and the eight values in figure 4 correspond to that amplitude. The ratio of half-axes b/a is defined as positive for rotation in the orbit, which is in the same sense as the spin of the ball. Whereas only two rates of spin were examined over a range of eccentricities, a wider range of spin rates was used for the case of linear swings ($b/a = 0$) (figure 5). The length of the pendulum, which had initially been 190 cm, was increased to 840 cm for the subsequent investigations, so reducing the relative magnitude of the corrections as far as the height of the building permitted.

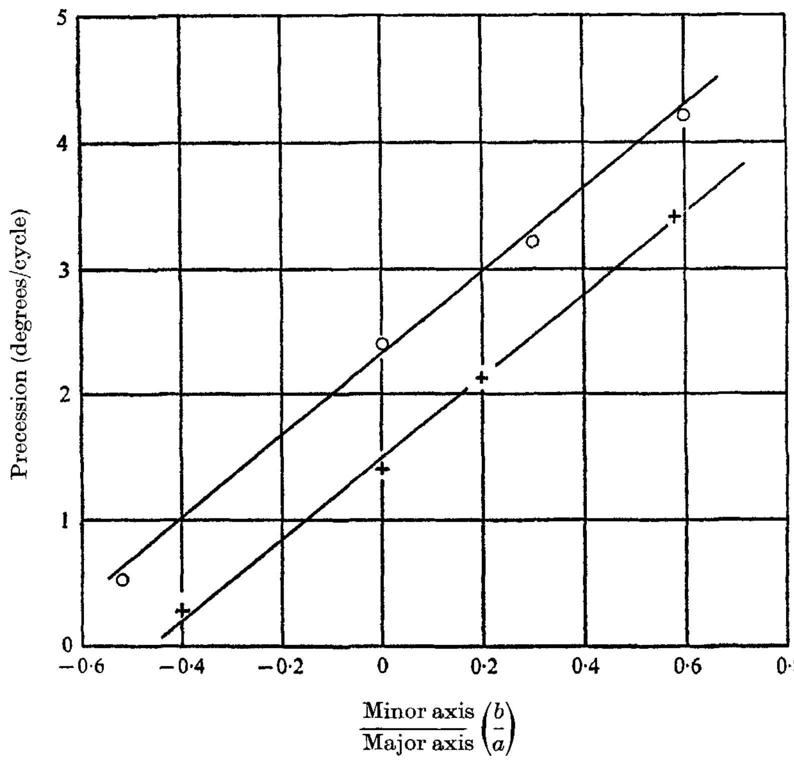

FIGURE 4. Rate of precession of the axes in relation to the eccentricity of the elliptical cone pendulum. Rate of rotation of sphere: O, 1440 rev/min; +, 1040 rev/min.

3.2. *Periodic times of the conical pendulum*

If the axis of spin of the sphere is assumed to be aligned with the string, and the aerodynamic lift L is taken to be normal to the string (figure 6), the ratio τ/τ_0 of the periodic time with spin to that without spin can be shown to be

$$\frac{\tau}{\tau_0} = \frac{1}{[1 - L/(mg\sin\theta)]^{\frac{1}{2}}}. \tag{1}$$

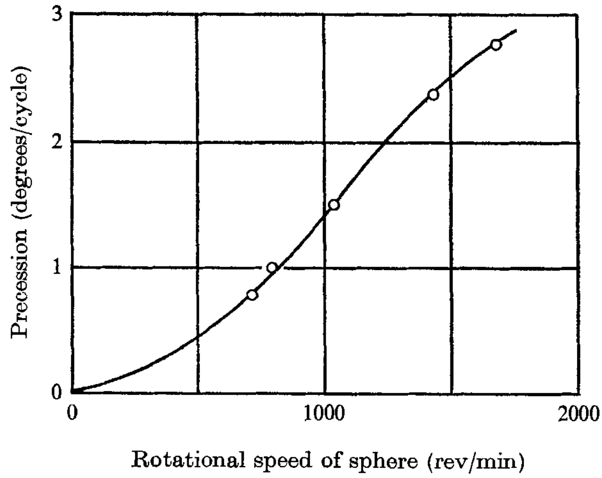

FIGURE 5. Rate of precession of straight pendulum swings, as a function of the rate of spin of the ball.

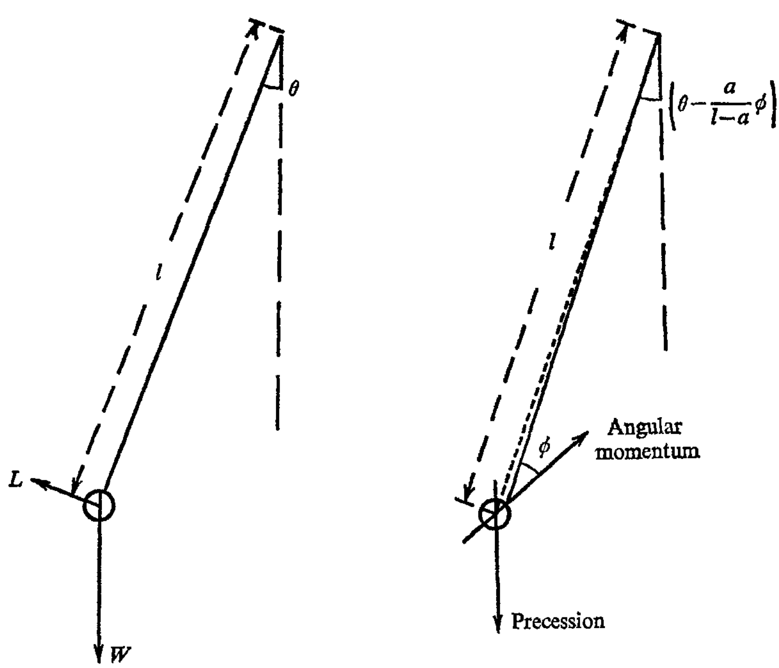

FIGURE 6 FIGURE 7

FIGURE 6. Conical pendulum with spinning spherical mass of negligible moment of inertia; the axis of spin is aligned with the suspension.

FIGURE 7. Conical pendulum with sphere of finite moment of inertia; the axis of rotation of the sphere assumes such an angle to the suspension as to cause a precession having a period equal to that of the pendulum.

To a good approximation the increase of period $(\delta\tau)_a = \tau - \tau_0$, due to the aerodynamic force in the presence of spin, may then be expressed as

$$\frac{(\delta\tau)_a}{\tau_0} = \frac{L}{2mg\sin\theta}. \tag{2}$$

Defining the coefficient of lift in terms of the projected area S of the ball, and the density ρ of the air,

$$L = \tfrac{1}{2}\rho C_L S U^2. \tag{3}$$

Now the translational velocity U in the circular orbit of angle θ is $[gl\sin\theta\tan\theta]^{\frac{1}{2}}$ when L is negligible, and using this in (3) and (2) the coefficient of lift can be expressed as

$$C_L = \frac{4m}{\rho Sl\tan\theta}\frac{(\delta\tau)_a}{\tau_0}. \tag{4}$$

The period is longer, and $(\delta\tau)_a$ positive, when the angular velocities of spin and orbit are opposed. Under the same condition the period would also exhibit a positive increment $(\delta\tau)_g$ due to gyroscopic action in the absence of any aerodynamic force. We will retain θ as the inclination to the vertical of the line from the centre of the ball to the point of suspension; then the angle of the string to the vertical is approximately $\theta - \{a/(l-a)\}\phi$, where ϕ is the angle between the axis of spin and the string (figure 7). The period without aerodynamic lift is related to τ_0 by

$$\frac{\tau}{\tau_0} = \left[\frac{\tan\theta}{\tan\,(\theta - \{a/(l-a)\}\phi)}\right]^{\frac{1}{2}}, \tag{5}$$

and the increment $(\delta\tau)_g$ due to gyroscopic effect is then given approximately by

$$\frac{(\delta\tau)_g}{\tau_0} = \frac{1}{\sin 2\theta}\frac{a}{l}\phi. \tag{6}$$

The angle of tilt of the ball in the steady state is such that the tension of the string, acting at distance $a\sin\phi$ from the centre of gravity of the ball, causes precession of the spin axis at the rotation rate of the conical pendulum. All the observations recorded here were made with $\theta < 5°$ and $\phi < 1°$, and the ratio a/l was $3{\cdot}6\times10^{-3}$, so that the relation between moment, angular velocity of precession and angular momentum is adequately represented by

$$\frac{\sin\phi}{\sin(\theta+\phi)\cos^{\frac{1}{2}}\theta} = \frac{2a}{5(gl)^{\frac{1}{2}}}\omega, \tag{7}$$

where ω is the angular velocity of the ball; the ball is assumed to be a homogeneous sphere. This was solved graphically to give the values of ϕ required in (6), and the gyroscopic contribution $(\delta\tau)_g$ to the observed period was thus calculated.

At latitude Λ the earth's rotation increases the period of a conical pendulum with anticlockwise motion (looking down) by

$$(\delta\tau)_e = 1{\cdot}16\times10^{-5}\times\tau^2\sin\Lambda. \tag{8}$$

$(\delta\tau)_e$ is simply reversed by reversal of the orbit, and $(\delta\tau)_a$ is reversed by reversal of either spin or orbit; the positive and negative values of $(\delta\tau)_g$, however, differ appreciably; $\overline{(\delta\tau)_g}$ is the mean of the two.

Since $(\delta\tau)_a$, $\overline{(\delta\tau)_g}$ and $(\delta\tau)_e$ are all very small compared to τ_0, it is justifiable to regard the observed $\overline{\delta\tau}$ as the sum of these, each being represented by one of the separate formulae (2), (6) and (8).

The corrections of the period for damping are negligible. In the most extreme case observed the damping was 1·4 % of critical, thus increasing the period 5·8 s by 0·6 ms. The mean damping was about $\frac{1}{2}$ % of critical, increasing the period by less than 0·1 ms, so that any differences, due to different damping for the two directions of rotation, were smaller still.

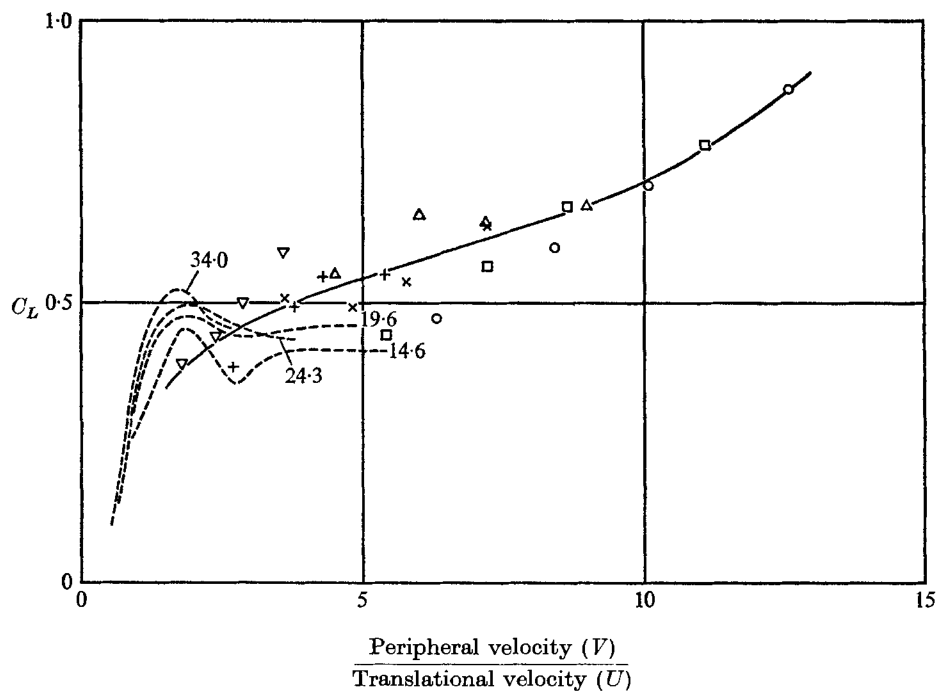

FIGURE 8. The relation of the coefficient of lift of a sphere to its relative rate of rotation, given by the periods of a conical pendulum. The broken lines are the results of Maccoll, who used a smooth ball of diameter 6 in.; each line is for one velocity of translation, indicated in ft s^{-1}. Rate of rotation of sphere: ○, 1400 rev/min; □, 1200 rev/min; △, 1000 rev/min; ×, 800 rev/min; +, 600 rev/min; ▽, 400 rev/min.

Observations of the period were all made with the ball rotating in one direction (clockwise, looking down), the orbital motion being reversed. Every second orbit was timed electronically, while the size of the orbit was observed through a telescope close to the point of suspension: circles subtending radial angles of 0·04, 0·05, 0·06 and 0·08 rad had been marked on a graticule. As the orbits frequently departed noticeably from true circles, judging when they matched the circles of the graticule was a major source of error. Individual periods deviated from the smoothed curve by up to 3 ms, but the smoothed figures are good to about 1 ms. Differences at any given cone angle between the smoothed periodic times with opposite orbital motions, double the $\overline{\delta\tau}$ of the above equations, ranged from 20 to 40 ms. The correction term $[(\delta\tau)_g + \overline{(\delta\tau)_e}]$ amounted to between

4 and 12 % of the observed figure for $\overline{\delta\tau}$, and the derived values for the lift coefficient C_L have a mean accuracy of 5 %. Translational Reynolds numbers lay in the range 1500–3000.

The plot in figure 8 of C_L against V/U, the ratio of peripheral and translational speeds, shows a trend to higher lift coefficients at lower values of either V or U; lines of constant V are, however, better defined than are those of constant U.

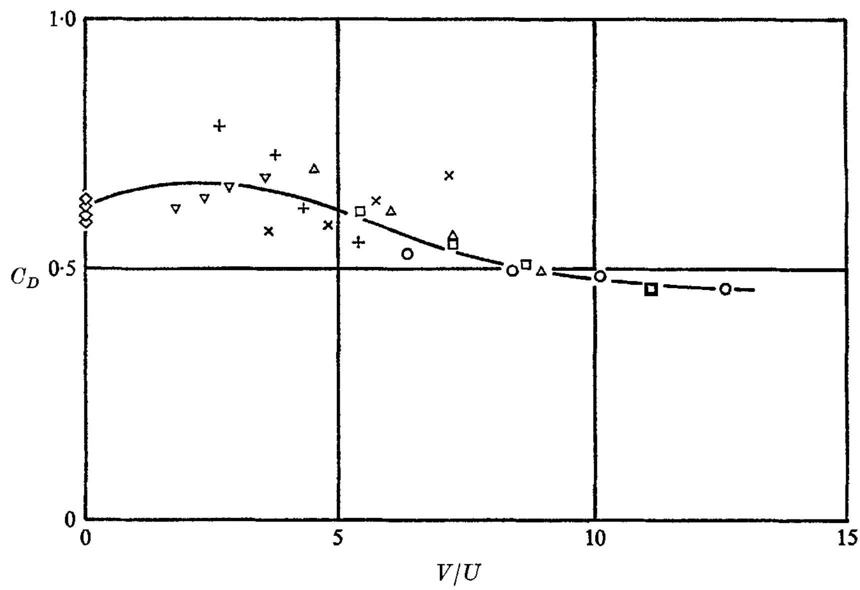

FIGURE 9. Drag coefficients derived from the rate of decay of conical orbits. Symbols as in figure 8 with addition ◇ for zero rate of rotation.

The drag coefficients of figure 9 exhibit no corresponding separation. These are calculated on the assumption that the resistance of the suspending filament is negligible in comparison with that of the sphere. The radius r of the orbit being recorded as a function of the number N of the cycle, the drag coefficient can then be shown to be given by the relation

$$C_D = \frac{m}{\pi \rho s} \frac{d(1/r)}{dN}. \tag{9}$$

The nylon filament had a length of 837 cm and a diameter of 0·35 mm, so that its total projected area was comparable to that of the sphere. Although no attempt has been made to evaluate its effect in detail, the added resistance accounts roughly for the drag shown for the sphere at zero spin exceeding the accepted figure by some 30 %.

In common with whirling-arm experiments in general, these measurements suffer from the defect that the ball is not moving into still air. The smallest orbit used had a diameter eleven times the ball diameter, and the period, the longest possible, was still under 6 s.

4. Conclusion

The coefficients of lift for rotating spheres at Reynolds numbers between 1500 and 3000, calculated to an accuracy of 5 % from differences in the periods of a conical pendulum, show noticeable departures from earlier measurements by Maccoll at Reynolds numbers of 10^5. Whereas Maccoll's lift coefficients level off at higher rates of spin, the new observations rise less rapidly at first, with increasing spin, but continue to rise to higher values, tending towards proportionality to the rate of spin. Though of lower absolute accuracy, the drag coefficients exhibit an appreciable difference from those of Maccoll, in that they fall slightly with increasing spin.

Dr A. D. D. Craik gave valuable advice throughout the work. The later observations of the conical pendulum were made with the assistance of Mr M. A. Barkla.

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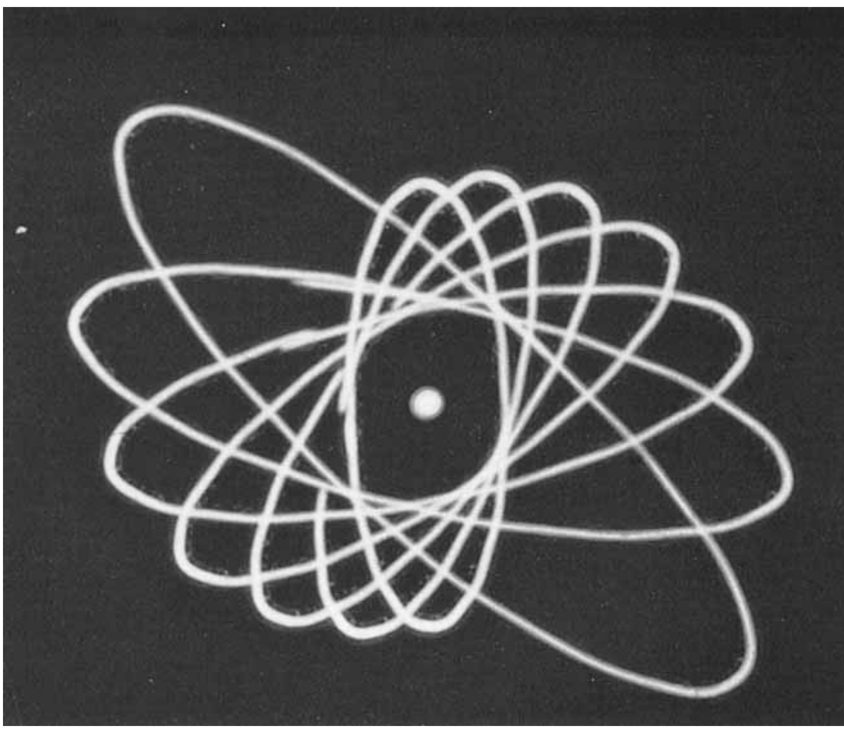

FIGURE 3. A typical record of the elliptical pendulum motion. Every fifth orbit was photographed. The ball was spinning at 1440 rev/min and the greatest amplitude shown is about 1/6 radian.